Best Arm Identification for Stochastic Rising Bandits

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Motivation

Several real-world scenarios can be formalized as a **Best-Arm Identification** (BAI) problem in the **Stochastic Rising Bandits** (SRB) setting:

- Combined Algorithm Selection and Hyperparameter Optimization (*CASH*)
- Best Model Selection
- Selection of Athletes for Competitions

Contributions

- Extension of the SRB setting to the **fixed-budget BAI** problem
- Setting **lower bound** on the error probability
- **Two algorithms** solving the problem:
  - `R-UCBE`: an optimistic algorithm
  - `R-SR`: a phase-based algorithm
- **Theoretical analysis** of the error probability upper bounds
- **Numerical validation** on synthetic and real-world data

Setting - Overview

Fixed Budget BAI for SRB

Reward	$x_t = \underbrace{\mu_{I_t}(N_{I_t,t})}_{\text{Expected reward}} + \underbrace{\eta_t}_{\text{Noise}}$
Budget	T
Best Arm	$i^*(T) := \arg\max_{i \in [\![K]\!]} \mu_i(T)$
Growth Rate	$\gamma_i(n) := \mu_i(n+1) - \mu_i(n)$

Rising Bandits

Non-Decreasing: $\gamma_i(n) \geqslant 0$

Concavity: $\gamma_i(n+1) \leqslant \gamma_i(n)$

Bounded Growth Rate

$\gamma_i(n) \geqslant cn^{-\beta}$

$c \geqslant 0$ and $\beta > 1$

Goal

Minimize

(1) Error Probability

$$e_T(\nu, \mathfrak{A}) := \mathbb{P}_{\nu,\mathfrak{A}}(\hat{I}^*(T) \neq i^*(T))$$

(2) Simple Regret

$$r_T(\nu, \mathfrak{A}) := \mu_{i^*(T)}(T) - \mu_{\hat{I}^*(T)}(N_{\hat{I}^*(T),T})$$

Setting - Lower Bound

Sub-optimality Gap $\Delta_i(T) := \mu_{i^*(T)}(T) - \mu_i(T)$

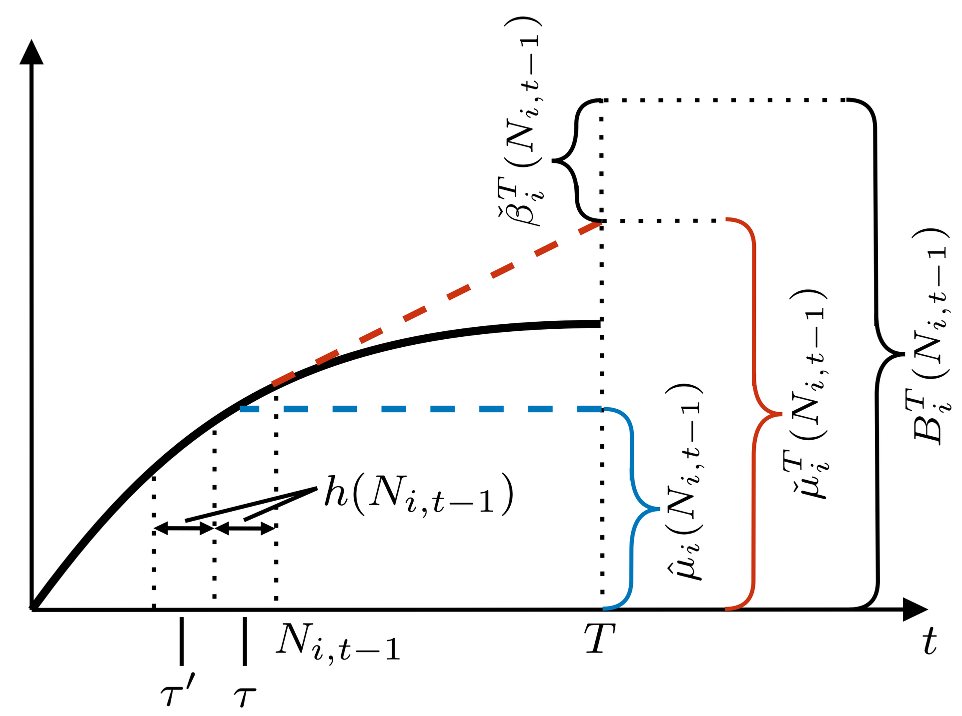
Error Probability

$$e_T(\mathfrak{A}) \geqslant \frac{1}{4}\exp\left(-\frac{8T}{\sigma^2 \sum_{i \neq i^*(T)} \frac{1}{\Delta_i(T)^2}}\right)$$

Time Budget

$$T \geqslant \sum_{i \neq i^*(T)} \left(\frac{1}{8\Delta_i(T)}\right)^{\frac{1}{\beta-1}}$$

Estimators



Pessimistic Estimator

$$\hat{\mu}_i(N_{i,t-1}) := \frac{1}{h(N_{i,t-1})} \sum_{\tau \in \mathcal{T}_{i,t}} x_\tau$$

Optimistic Estimator

$$\check{\mu}_i^T(N_{i,t-1}) := \hat{\mu}_i(N_{i,t-1}) + \sum_{(\tau,\tau') \in \mathcal{S}_{i,t}} (T-j)\frac{x_\tau - x_{\tau'}}{h(N_{i,t-1})^2}$$

Window Selection

$$h(N_{i,t}) = \lfloor \varepsilon N_{i,t} \rfloor \qquad \varepsilon \in (0, 1/2)$$

Algorithms

R-UCBE

Input: T, K, ε, a
Initialize $N_{i,0} = 0$, $B_i^T(0) = +\infty, \forall i \in [\![K]\!]$
for $t \in [\![T]\!]$ **do:**
  Compute $I_t \in \arg\max_{i \in [\![K]\!]} B_i^T(N_{i,t-1})$
  Pull arm I_t and observe x_t
  Update $\check{\mu}_{I_t}^T(N_{I_t,t})$ and $\check{\beta}_{I_t}^T(N_{I_t,t})$
  $B_{I_t}^T(N_{I_t,t}) = \check{\mu}_{I_t}^T(N_{I_t,t}) + \check{\beta}_{I_t}^T(N_{I_t,t})$
end
Recommend $\hat{I}^*(T) \in \arg\max_{i \in [\![K]\!]} B_i^T(N_{i,T})$

R-SR

Input: T, K, ε
Initialize $t \leftarrow 1$, $N_0 = 0$, $\mathcal{X}_0 = [\![K]\!]$
for $j \in [\![K-1]\!]$ **do:**
  for $i \in \mathcal{X}_{j-1}$ **do:**
    for $l \in [\![N_{j-1}+1, N_j]\!]$ **do:**
      Pull i and observe x_t, $t \leftarrow t+1$
    end
    Update $\hat{\mu}_i(N_j)$
  end
  $\mathcal{X}_j = \mathcal{X}_{j-1} \backslash \{\bar{I}_j\}$ with $\bar{I}_j \in \arg\min_{i \in \mathcal{X}_{j-1}} \hat{\mu}_i(N_j)$
end
Recommend $\hat{I}^*(T) \in \mathcal{X}_{K-1}$ (unique)

Theoretical Guarantees

R-UCBE:

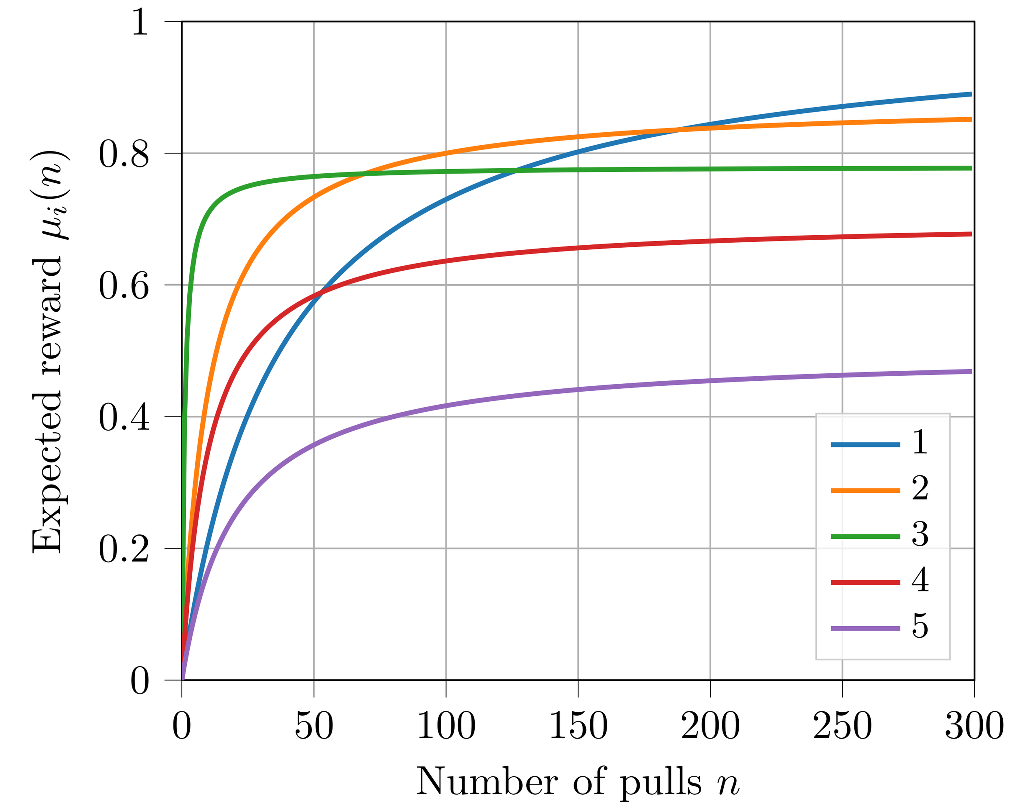
e_T	$2TK\exp\left(-\frac{a}{10}\right)$
r_T	$e_T + c4^\beta T^{1-\beta}$
T	$2\left(c^{1/\beta}(1-2\varepsilon)^{-1}H_{1,1/\beta}(T) + K - 1\right)^{\frac{\beta}{\beta-1}}$ if $\beta \in (1, 3/2)$
	$2\left(c^{2/3}(1-2\varepsilon)^{-2\beta/3}H_{1,2/3}(T) + K - 1\right)^3$ if $\beta \geqslant 3/2$

R-SR:

e_T	$\frac{K(K-1)}{2}\exp\left(-\frac{\varepsilon}{8\sigma^2}\cdot\frac{T-K}{\overline{\log}(K)\max_{i\in[\![2,K]\!]}\{i\Delta_{(i)}(T)^{-2}\}}\right)$
r_T	$e_T + c4^\beta T^{1-\beta}\overline{\log}(K)^\beta$
T	$2^{\frac{\beta+1}{\beta-1}}c^{\frac{1}{\beta-1}}(1-\varepsilon)^{-\frac{\beta}{\beta-1}}\overline{\log}(K)^{\frac{\beta}{\beta-1}}\max_{i\in[\![2,K]\!]}\left\{i\Delta_{(i)}(T)^{-1/\beta}\right\}^{\frac{\beta}{\beta-1}}$

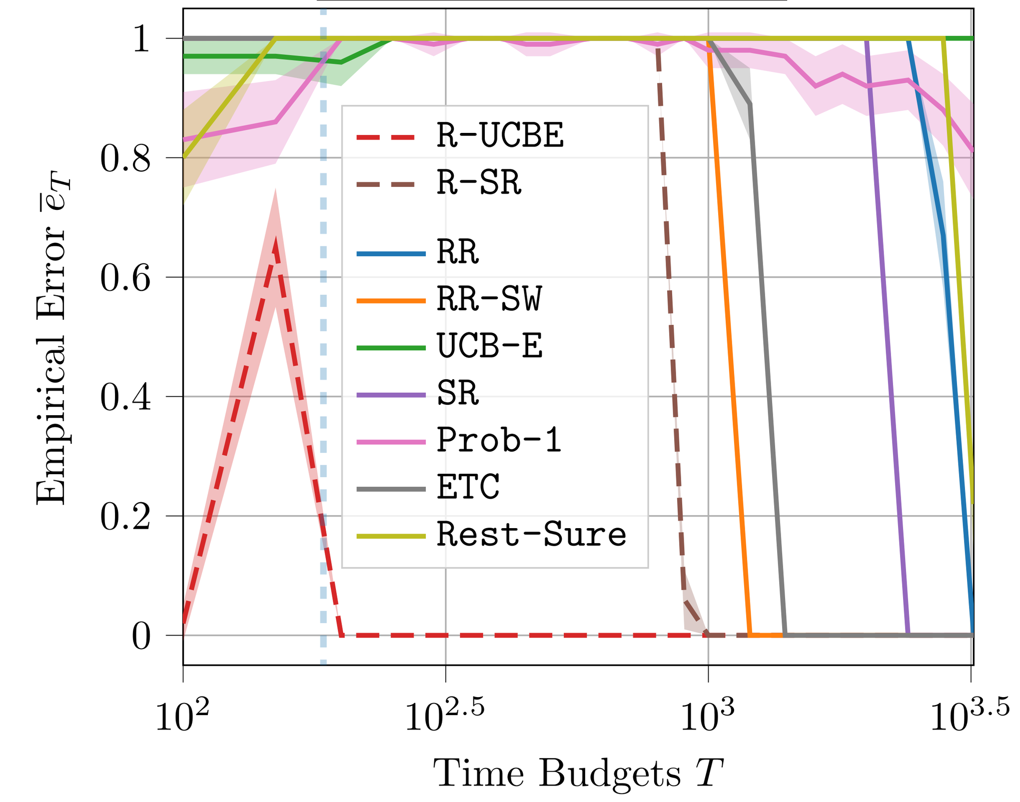
Experimental Validation

Setting

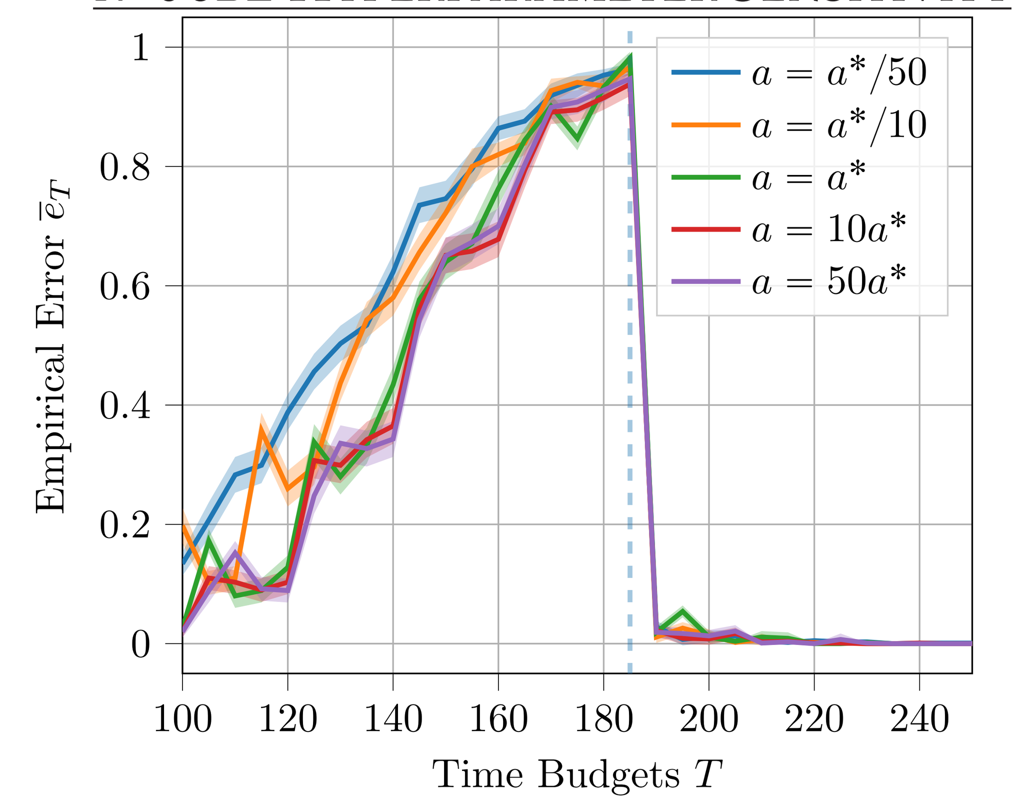


Results

Error Probability



R-UCBE Hyperparameter Sensitivity



References

Jean-Yves Audibert, Sébastien Bubeck, and Rémi Munos. Best arm identification in multi-armed bandits. In *COLT*, 2010.

Alberto Maria Metelli, Francesco Trovò, Matteo Pirola, and Marcello Restelli. Stochastic rising bandits. In *ICML*, 2022.