POLTECNICO
DY
YNAMIC PRICING WITH VOLUME DISCOUNTS IN ONLINE SETTINGS
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$\underbrace{}_{\text {DYNAMIC PRICING }}$

## WHY THIS WORK?

- E-commerce websites may face different kinds of users, both customers and businesses.
- Different kinds of users present different needs in terms of item quantity.

- The kind of user the website will face is not known a priori. - We want to propose different prices for different volume thresholds thanks to the introduction of discounts.


## SETTING AND Goal

## SETTING

- We want to price the products independently.
- We consider a setting in which only transaction data are available.
- The demand curve is assumed to be monotonic in price.

GOAL
At each time $t$, find:

- a set of volume thresholds $\omega_{t}:=\left[\omega_{1 t}, \ldots, \omega_{\eta t}\right] \in \mathbb{N}^{\eta}$ - a set of prices $\boldsymbol{p}_{t}:=\left[p_{1 t}, \ldots, p_{\eta t}\right] \in \mathcal{P}^{\eta}$


The goal is to select sets $\omega_{t}$ and $p_{t}$ maximizing the total profit $R(T)$ over time horizon $T$ :

$$
R(T):=\sum_{t=1}^{T} \sum_{i=1}^{\eta}\left(p_{i t}-c\right) \cdot v_{i}\left(\boldsymbol{p}_{t}, \boldsymbol{\omega}_{t}, t\right)
$$

## ALGORITHM

IDEA: If we face this problem using a single machine learning model, the learning complexity scales exponentially in the number of thresholds, and cannot be addressed effectively in the presence of scarce data.
We decompose the problem of finding both volume thresholds and corresponding optimal prices in two sub-problems:

1. Find the optimal average price for a given product without considering volume discounts.
2. Build an adaptation scheme to obtain different prices for different volumes, whose (weighted) average is equal to the optimal price previously obtained.


## Optimal Price Estimation

- We want to find the optimal price $p_{t}^{*}$ for every product independently
- For each $t$ in the past, we compute the total volumes $\bar{v}_{t}$ and the average price $\bar{p}_{t}$
- Volume curves $\hat{v}(p, t)$ are generalized using Bayesian Linear Regression from price ( $p$ ) and time $(t)$ related features to generalize over demand curve and seasonality
- Price-related features are selected to be monotonic nonincreasing, their weight distributions are chosen with a positive support
- At a given time $\tau$, we can fix time-related feature, and sample using a Thompson Sampling-like approach a curve $\hat{v}_{T S}(p, \tau)$ binding pricing and volumes
- We estimate the best average price $p_{\tau}^{*} \in \underset{p \in \mathcal{P}}{\arg \max }(p-c) \cdot \hat{v}_{T S}(p, \tau)$


Real-World Application

## REAL-WORLD SETTING

$\mathrm{A} / \mathrm{B}$ test involving $\approx 300$ products with $\approx 300$ kEuros of turnover and $\approx 83$ kEuros of margin. The test is conducted for 4 months in summer/fall 2021. The e-commerce specialists ask us to condsider $\eta=3$ thresholds.


We observe that volume discounts are fundamental, given that most of the customers are not loyal

Results Overview
We improve the performance of set A of $55 \%$ w.r.t. the set B


We register in set A the $47 \%$ of products which increse their turnover, against the $25 \%$ of set B

Volume Discounts Effect
The average discount applied for the algorithm are $10 \%$ for the second volume interval and $20 \%$ for the third one.

| Product | $\Delta \bar{\beta}_{1}$ | $\Delta \bar{\beta}_{2}$ | $\Delta \bar{\beta}_{3}$ | $\\|$ units |
| :---: | :---: | :---: | :---: | :---: |
| 1 | $-32 \%$ | $+10 \%$ | $+22 \%$ | $+63 \%$ |
| 2 | $-26 \%$ | $+25 \%$ | $+1 \%$ | $+43 \%$ |
| 3 | $-15 \%$ | $+4 \%$ | $+11 \%$ | $+11 \%$ |
| 4 | $-5 \%$ | $+1 \%$ | $+4 \%$ | $+14 \%$ |
| Mean | $-19.5 \%$ | $+10 \%$ | $+9.5 \%$ | $+33 \%$ |

CONCLUSIONS AFTER THE A/B TEST

- After the $\mathrm{A} / \mathrm{B}$ test, the e-commerce website decide to adopt the solution
- The dynamic pricing algorithm is now pricing over 1200 products for a total turnover of 1.5 MEuros.


## References

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