Dynamic Pricing with Volume Discounts in Online Settings

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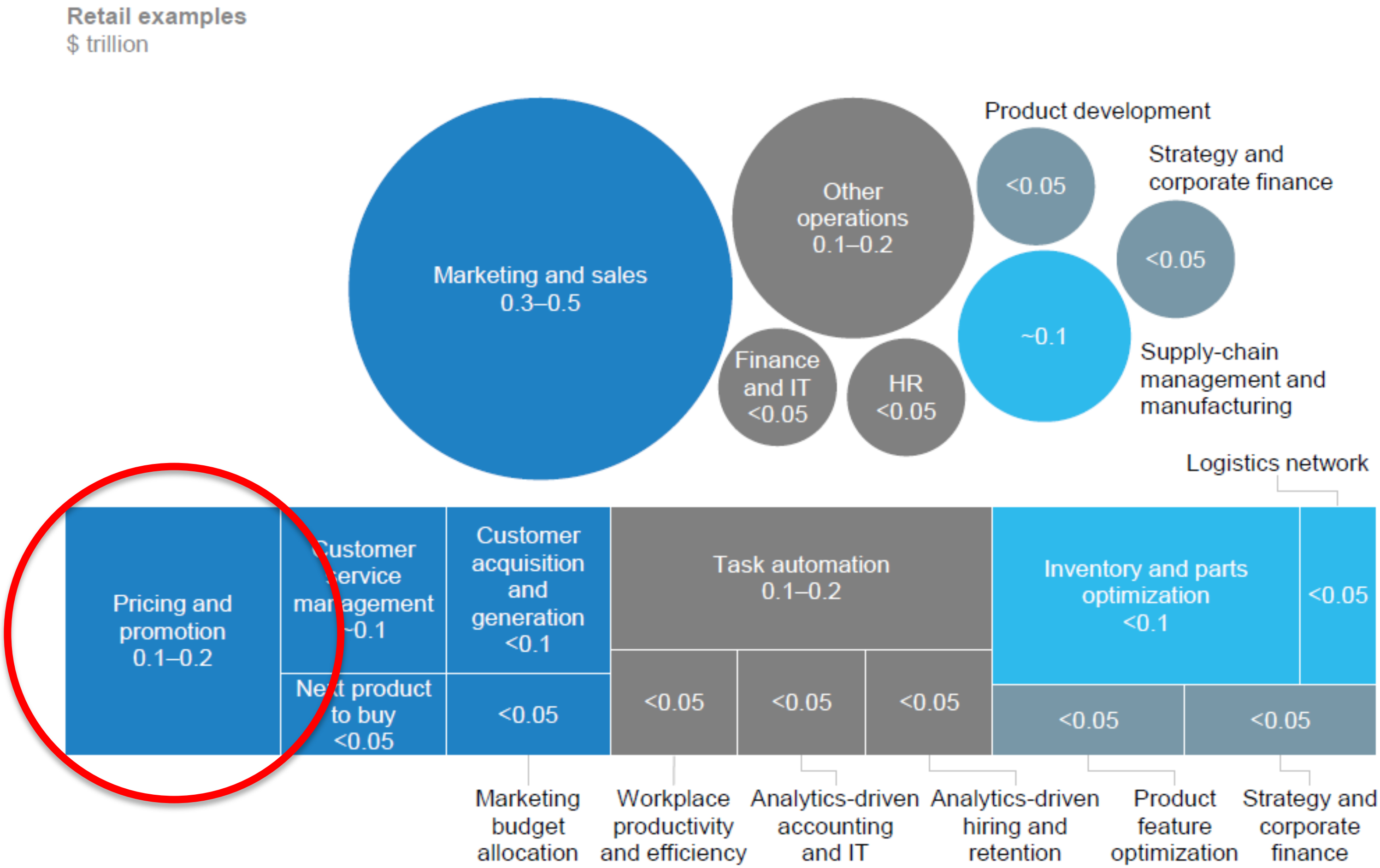
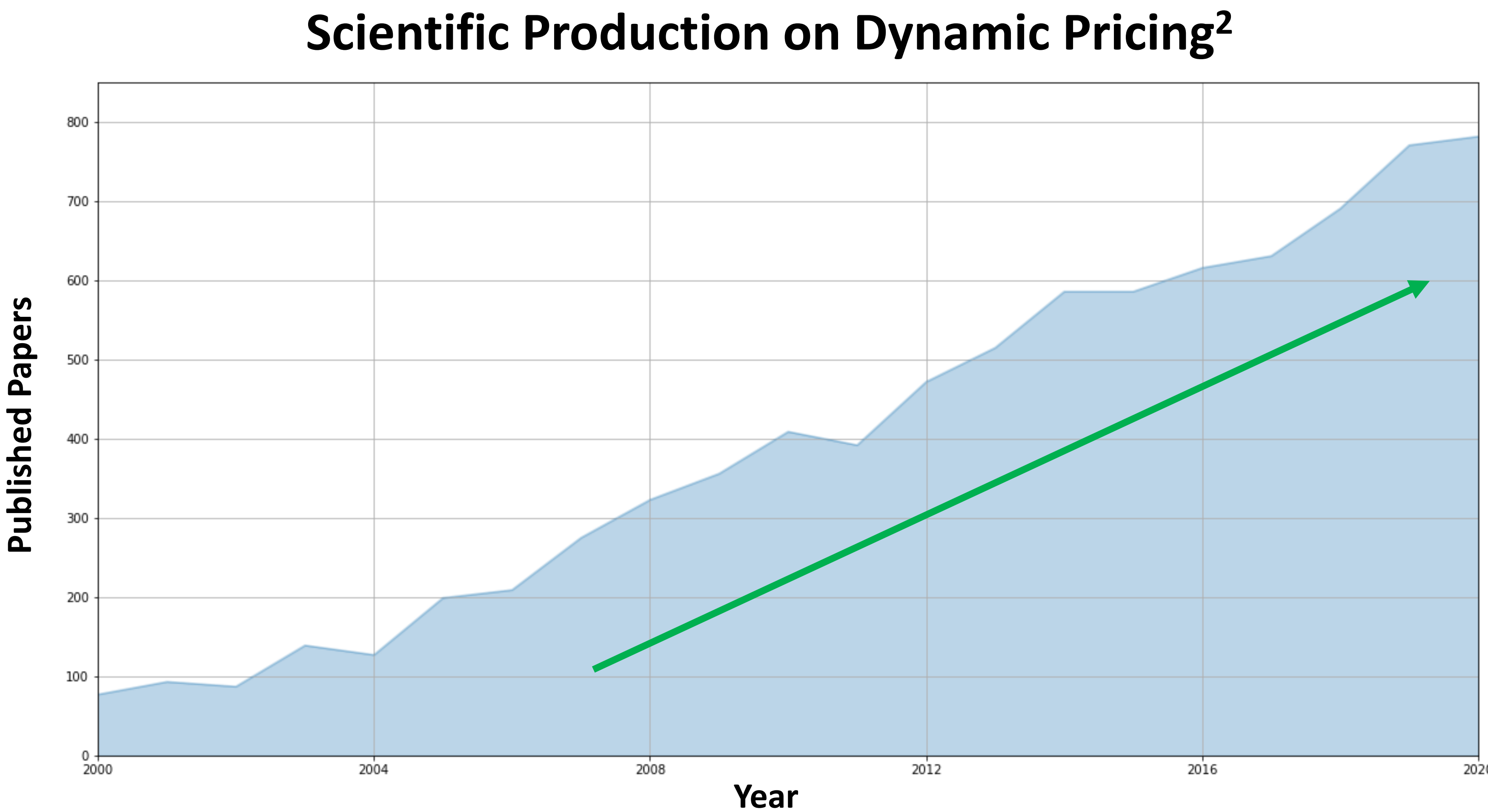
35th IAAI Conference, Washington D.C.

AI in Retail Business



[1] Chui, Michael, et al. "Notes from the AI frontier: Insights from hundreds of use cases." *McKinsey Global Institute* (2018).

Research on Dynamic Pricing

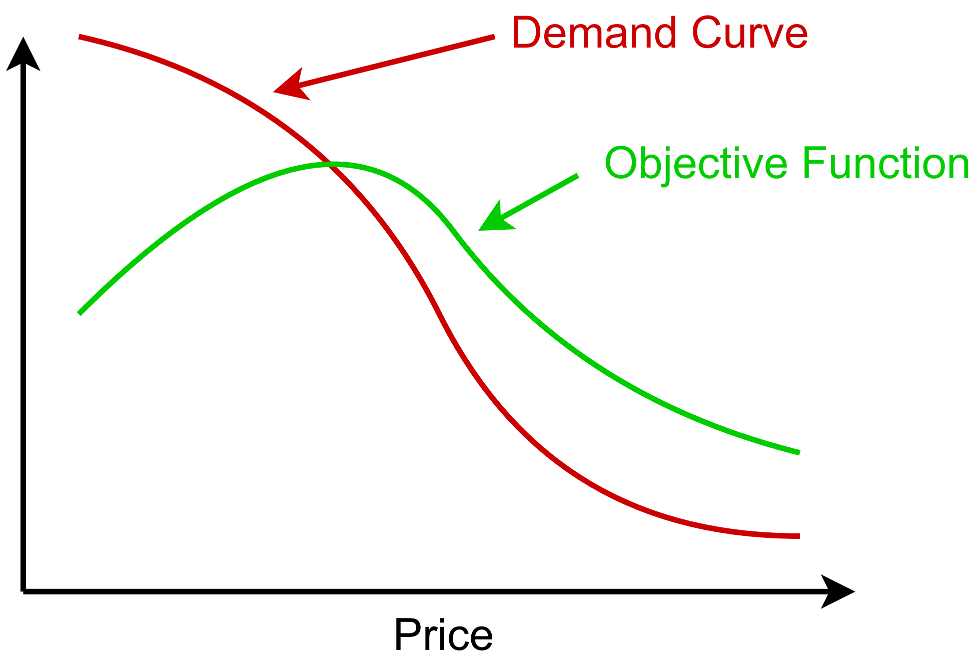


[2] www.scopus.com

Dynamic Pricing

Consider an e-commerce which sells a product

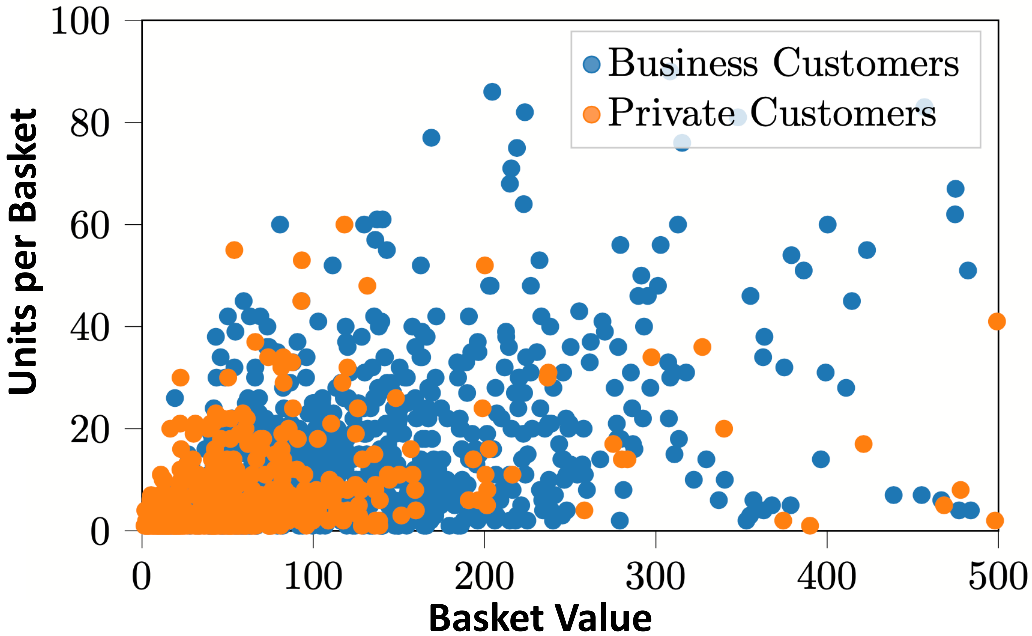
- Customers **visit** the product page and **decide** whether to buy or not
- By aggregating users choices, we are able to build a **demand curve**



[3] Arnoud V Den Boer. Dynamic pricing and learning: historical origins, current research, and new directions. Surveys in operations research and management science, 20(1):1–18, 201

Motivation

- E-commerce websites may face **different kind of users**, both customers and business (**not known before the sale**)
- Different kind of users present different needs in terms of number of units to purchase

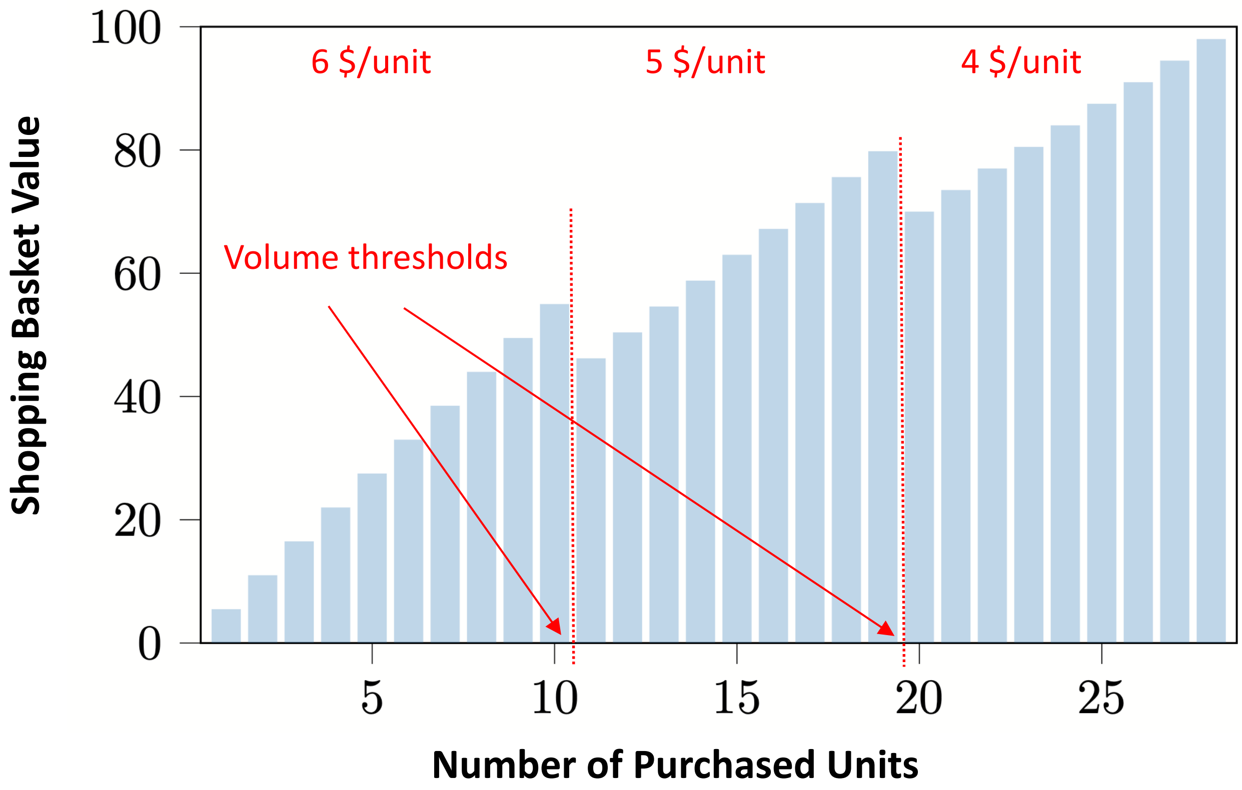


Goal

Provide a robust dynamic pricing algorithm allowing an e-commerce to face different kinds of users, without knowing their nature before the sale

Solution Idea

Offer customers volume discounts policies (a.k.a. quantity discounts) where **total expense is piecewise linear w.r.t. to units sold**



Setting and Goal

Problem Formulation – Assumptions

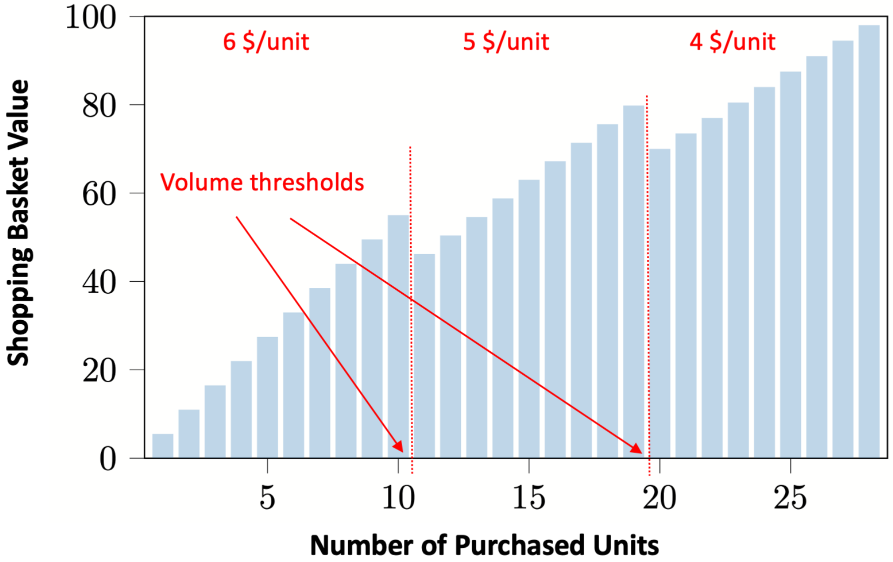
- We only consider goods that are **non-luxury**, so their **demand curve** is **decreasing in price**
- We assume that there is **no market interaction** between the products: **every product can be priced independently**
- **Only transaction data are available:**
  - Weekly Sales
  - Price History

Problem Formulation – Goal

W.l.o.g. we will consider the problem of **pricing a single good**

At every time t, find:

- a set of volume thresholds $\boldsymbol{\omega}_t := [\omega_{1t}, \ldots, \omega_{\eta t}] \in \mathbb{N}^\eta$
- a set of prices $\boldsymbol{p}_t := [p_{1t}, \ldots, p_{\eta t}] \in P^\eta$



Problem Formulation – Goal

Maximizing the **total profit** $R(T)$ over time horizon T:

$$R(T) := \sum_{t=1}^{T} \sum_{i=1}^{\eta} (p_{it} - c) \cdot v_i(\boldsymbol{p}_t, \boldsymbol{\omega}_t, t)$$

where c is the acquisition cost

Solution

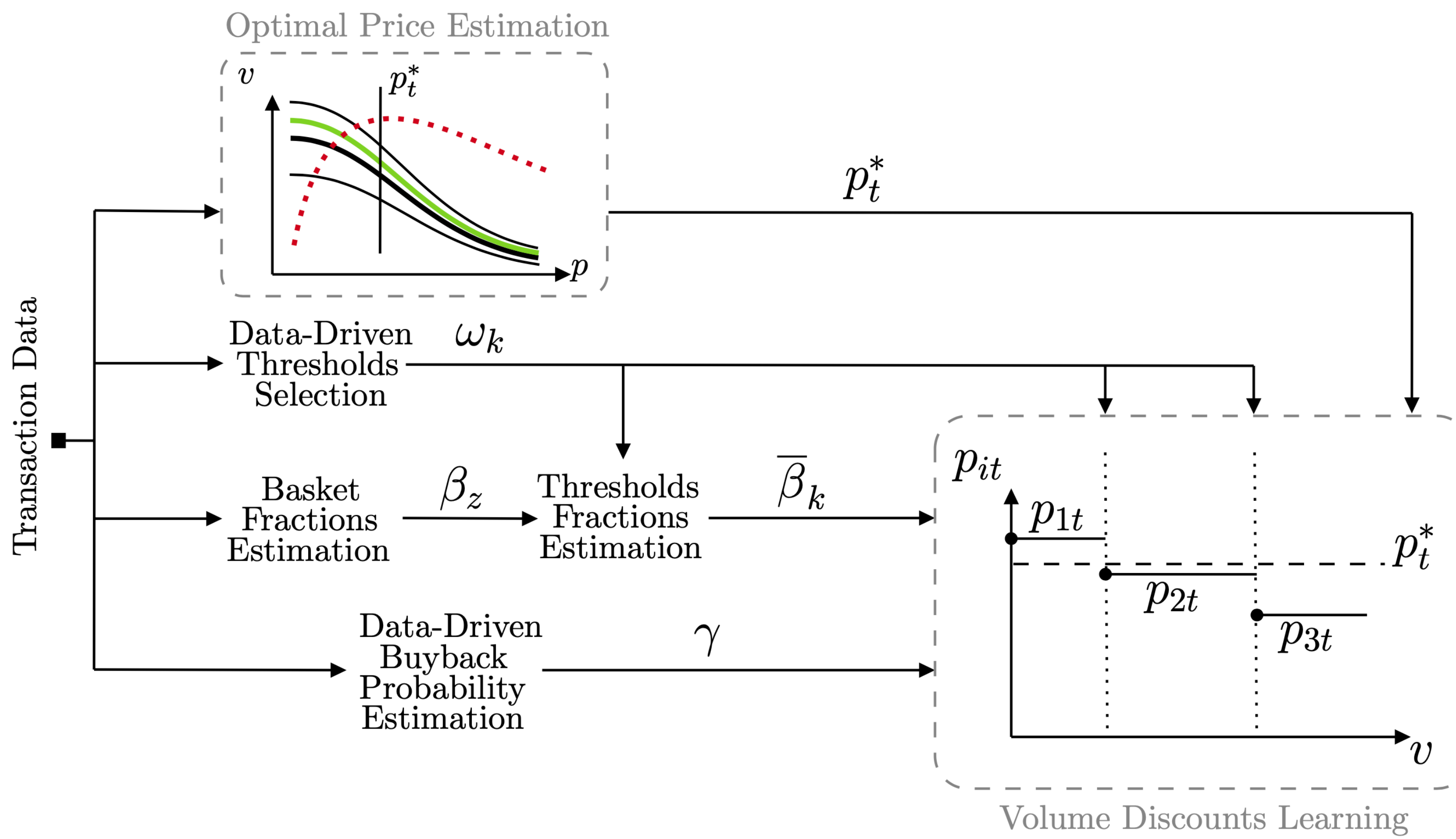
Naïve solution

- Design **a single, supervised ML model** that determines both the volume thresholds and the corresponding prices per unit to show
- The learning complexity scales **exponentially** in the number of thresholds
- **Not feasible in the presence of scarce data**.

Our Algorithm - Idea

- We **decompose** the problem of finding both volume thresholds and corresponding optimal prices in **two sub-problems**:
  1. Find the **optimal average price** for a given product **without taking into account volume discounts**
  2. Build an **adaptation scheme** to obtain different prices for different volumes, whose (weighted) mean is equal to the optimal price previously obtained

Algorithm - Overview



Solution

1 Optimal Average Price Estimation

Demand Curve Model

- We want to find the optimal average price p_τ^* at given time τ

- For each timestamp t in the past we compute, using transaction data, the total sales $\bar{v}_t$ and the average selling price $\bar{p}_t$

Demand Curve Model

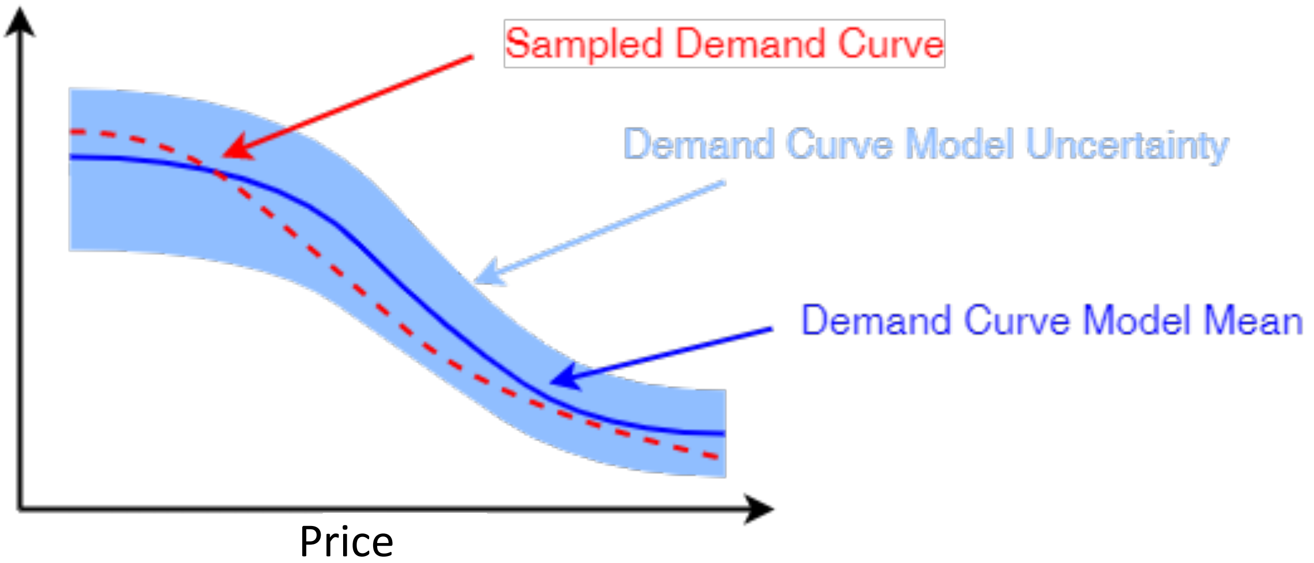
- The volume curve $\hat{v}(p, t)$ is modelled using a **Bayesian Linear Regression** that include features obtained from price p and time t

- Price-related features are obtained using **non-increasing basis functions**, while their weights are enforced to be positive to obtain **model's downward monotonicity w.r.t. price**

- Time-related features are obtained using polynomial and periodic basis functions to **account for seasonalities and trends in the market**

Dealing with Price Exploration

At a given time τ, we can fix time-related feature, and sample using a **Thompson Sampling-like** approach a curve $\hat{v}_{TS}(p, \tau)$ binding pricing and volumes

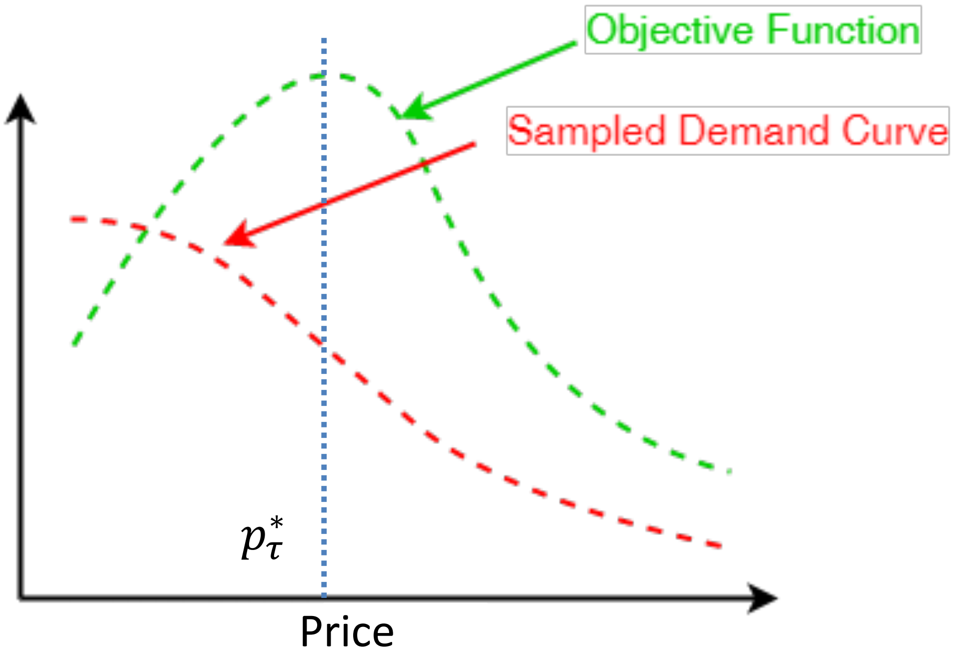


[5] Thompson, William "On the likelihood that one unknown probability exceeds another in view of the evidence of two samples." *Biometrika* (1933): 285-294.

Dealing with Price Exploration

We estimate the best average price:

$$p_\tau^* \in \arg\max_{p \in P} (p - c) \cdot \hat{v}_{TS}(p, \tau)$$



[5] Thompson, William "On the likelihood that one unknown probability exceeds another in view of the evidence of two samples." *Biometrika* (1933): 285-294.

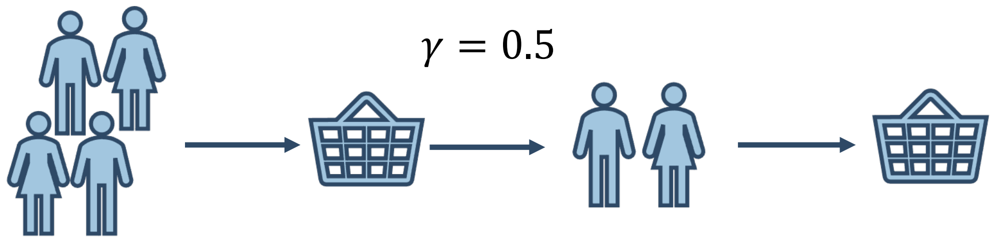
Solution

② *Volume Discounts*

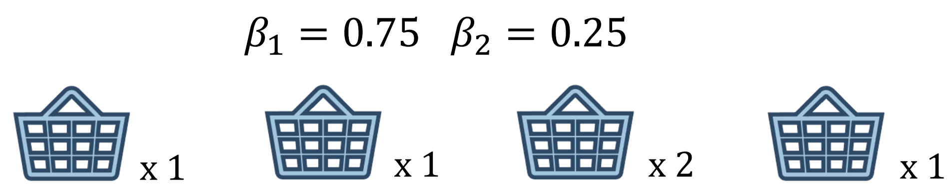
Extract knowledge from data

We extract quantitative insights from **transaction data** to characterize users' behavior

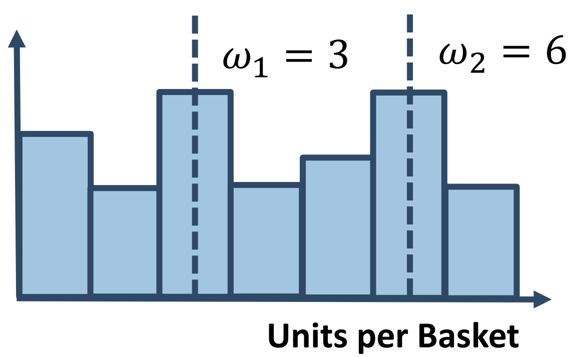
γ — Customers' buyback probability



β_z — % of baskets with z units of the product.

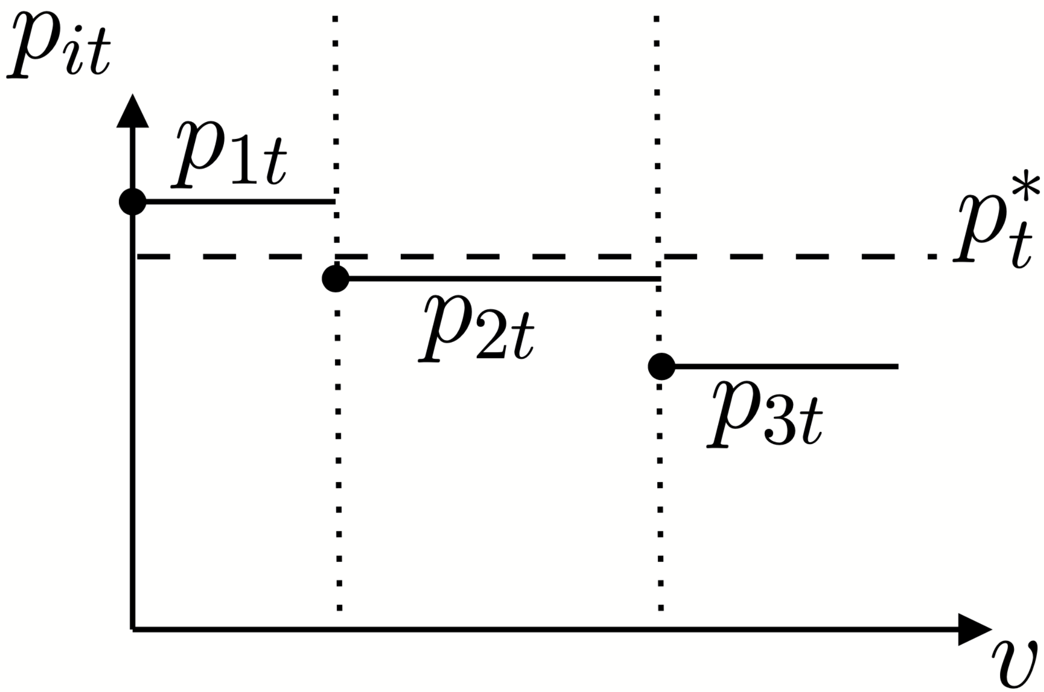


ω_k — Empirical quantity thresholds (from quantiles)



Multi-price pricing policy

We are able to obtain a **policy composed of multiple prices**, guaranteeing that their weighted (on the number of units sold) **average corresponds to the optimal price in output from Step 1**



Volume Discounts in Online Settings

- This procedure **combines data science techniques with mathematical modelling**

- It accounts for **online updates of the analyzed dataset**, since the pricing policy may vary from time to time depending on the market reaction to previous choices

Experimental Campaign

Experimental Setting

- The solution is tested in a **real** environment
- An A/B test is performed over 328 products with an yearly turnover of 300KEuros
- The test lasted for **4 months**

Configuration A (Test)

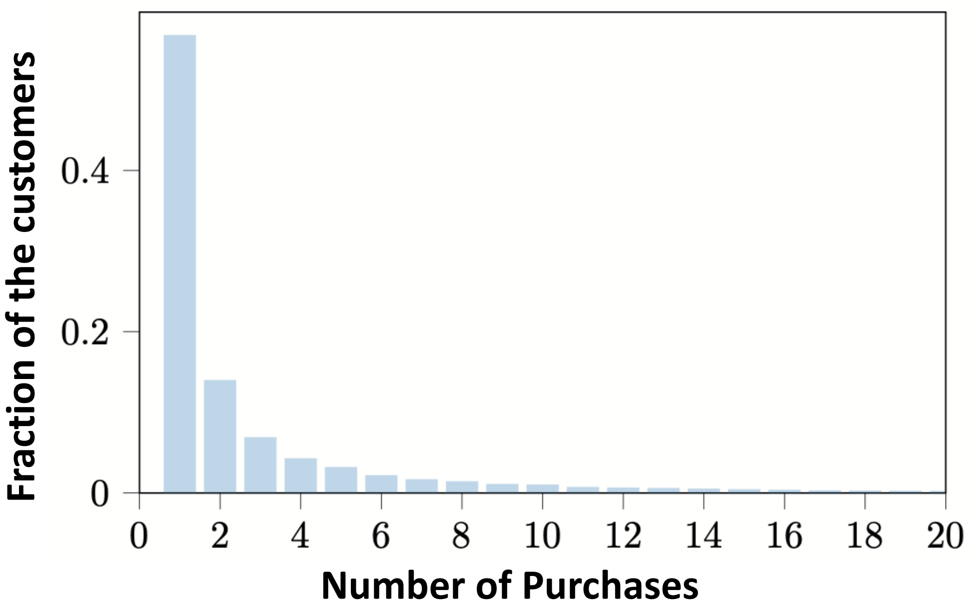
Priced by our algorithm

Configuration B (Control)

Priced by human specialist

Experimental Setting

- In this setting, revenue comes from both customers purchasing a few times and then leaving and customers that buy many times
- A pricing policy with volume discounts allows us to **maximize profits on these two categories of customers**



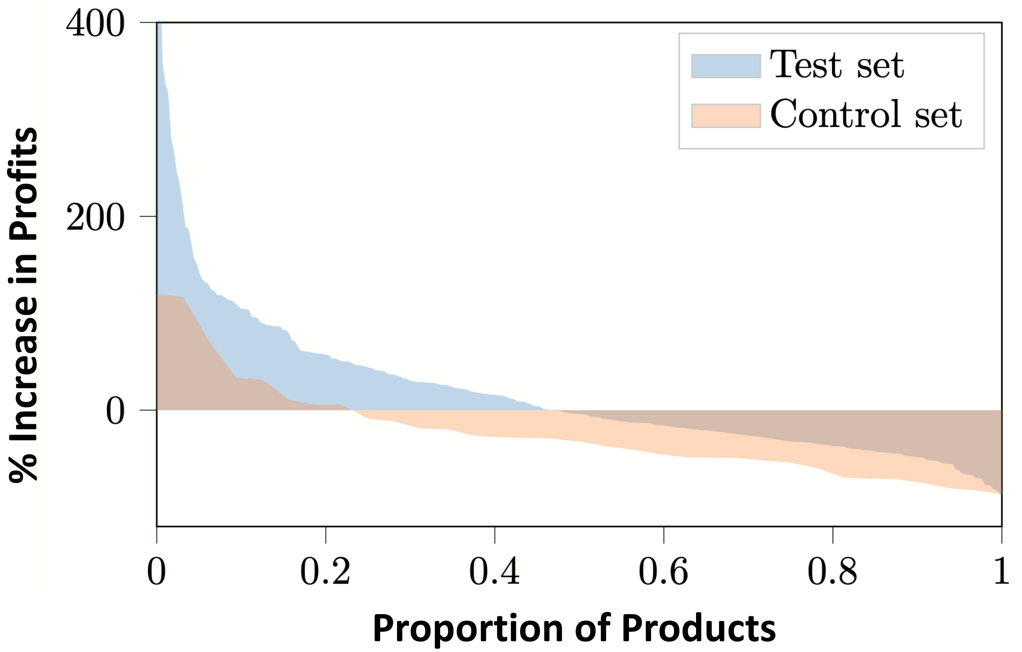
Evaluation Metric

The goal is to select sets $\boldsymbol{\omega}_t$ and $\boldsymbol{p}_t$ **maximizing** the **total profit** $R(T)$ over time horizon T:

$$R(T) := \sum_{t=1}^{T} \sum_{i=1}^{\eta} (p_{it} - c) \cdot v_i(\boldsymbol{p}_t, \boldsymbol{\omega}_t, t)$$

Results

Results are in favor of our algorithm, with observed profits that are **55% higher in the test configuration**



% of products improving their profits:

Configuration A (Test)
47%

Configuration B (Control)
25%

Effect of the Volume Discounts

The pricing specialists asked us to consider 3 **volumes thresholds**

Product	$\Delta\bar{\beta}_1$	$\Delta\bar{\beta}_2$	$\Delta\bar{\beta}_3$	Δunits
1	-32%	+10%	+22%	+63%
2	-26%	+25%	+1%	+43%
3	-15%	+4%	+11%	+11%
4	-5%	+1%	+4%	+14%
Mean	-19.5%	+10%	+9.5%	+33%

After the test, we observed an overall increment in purchases containing a number of units **above the second and/or the third volume thresholds**

Considerations After the A/B Test

- After the A/B test, the e-commerce website decided to adopt the solution **on its whole catalog**
- The algorithm is now pricing over 1200 products for a total turnover of 1.5 MEuros

Conclusions

- We provided the **first data-driven** dynamic pricing algorithm handling **volume discounts**
- We proposed a solution able to different **kinds of users** without having prior information about the type of user we are facing
- We evaluated the methodology through a **real-world campaign**, obtaining results in favor of the algorithm

Future Works

- Integration of **advertising** strategies in the presented pricing framework

- Modelling of products’ **interactions** in term of units sold

Thank you for the attention!

Take a look at our work

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